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Density of states of a two-dimensional electron gas in a perpendicular magnetic field and a random field of arbitrary correlation

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Abstract

A theory is given of the density of states (DOS) of a two-dimensional electron gas subjected to a uniform perpendicular magnetic field and any random field, adequately taking into account the *realistic correlation function* of the latter. For a random field of any long-range correlation, a semiclassical non-perturbative path-integral approach is developed and provides an analytic solution for the Landau level DOS. For a random field of any arbitrary correlation, a computational approach is developed. In the case when the random field is smooth enough, the analytic solution is found to be in very good agreement with the computational solution. It is proved that there is not necessarily a universal form for the Landau level DOS. The classical DOS exhibits a symmetric Gaussian form whose width depends merely on the rms potential of the random field. The quantum correction results in an asymmetric non-Gaussian DOS whose width depends not only on the rms potential and correlation length of the random field, but the applied magnetic field as well. The deviation of the DOS from the Gaussian form is increased when reducing the correlation length and/or weakening the magnetic field. Applied to a modulation-doped quantum well, the theory turns out to be able to give a quantitative explanation of experimental data with no fitting parameters.

1. Introduction

Since the discovery of the quantum Hall effect [1] the properties of a disordered two-dimensional electron gas (2DEG) subjected to a perpendicular magnetic field have been extensively studied. The nature of the density of states (DOS) of the 2DEG is a problem of vital importance for the understanding of many quantum phenomena observed in the system, e.g., cyclotron resonance, specific heat, magnetization, magnetocapacitance and magnetotransport.

The problem has attracted a lot of attention from both experimentalists and theorists [2]. Nevertheless, over the years of research the common conclusion has seemed to be only that the DOS in question is composed of disorder-broadened Landau levels with a significant number of electronic states in between. To date, there has been little or no consensus regarding the exact form of the Landau level DOS of the 2DEG and the magnetic-field dependence of the linewidth. This conclusion belongs not only to the calculated DOS but the measured DOS as well. Indeed, some experimentalists have reported a Gaussian lineshape [3–9], while others claim that it is Lorentzian [10, 11]. Some report a broadening which is independent of the applied magnetic field [3, 6, 10], while others report a broadening which is a square root [4, 5, 8] or oscillating [7, 12, 13] function of the magnetic field. In addition, in order to explain their experimental results several authors [3, 5, 8] have invoked a constant background DOS whose origin is unclear.

The first calculation of the Landau level DOS was performed within a self-consistent Born approximation (SCBA) [14–16]. However, based on a single-site picture, the SCBA theory underestimates the disorder effect. Within the many-site picture, the random field is characterized by a correlation function. This is shown [2, 17] to capture microscopic details of the electron system, e.g. the actual origin of the disorder as well as the actual geometry of the sample. Moreover, for true 2DEGs this is found to exhibit a very complicated dependence on its variables in real space. Therefore, in the existing literature one has to adopt a severe approximation, replacing the true spatial correlation of a random field by a simple one with some fitting parameters. The simplest form is a δ-function, which enables an exact solution to be found for the lowest Landau level DOS [18]. However, this model describes a zero-range electron–impurity interaction and none of the theories with this white-noise limit [19–22] predict a remarkable DOS lying between Landau levels.

Thus, the key problem is to keep the correlation length of the disorder finite from the outset [22–32] by assuming a Gaussian or exponential correlation function. Unfortunately, this simplification cannot allow an exact DOS solution and various approximation schemes have to be proposed. The Landau level DOS was evaluated by means of a cumulant expansion [22, 24, 25] for the Green function, or an expansion in the inverse correlation length for different quantities of interest, e.g. the Green function [28], the partition function [29] and the correlation function [30–32]. The validity of the approximations was often insufficiently discussed and, instead, the disorder was in practice considered to be a perturbation, as in the SCBA [22, 24, 30]. In addition, the theoretical prediction was found not to be in quantitative agreement with experimental findings [32].

The fact that so many different experiments seemingly yield so many different results for the Landau level DOS suggests that its exact form is very likely fixed by the very realistic nature of the 2DEG under consideration, i.e. by the experimental conditions in which the 2D electrons have been created and move. This means that, in order to quantitatively describe the measured Landau level DOS and its width, one has to work with the true correlation function. Recently, we have developed semiclassical approaches to a 1D and 2D electron in the presence of a random field of any long-range correlation [33, 34]. These turn out to meet that demand and to result in an analytic DOS solution. In the present paper, we will extend our methods to incorporate also a high perpendicular magnetic field into the theory.

It should be noted that semiclassical calculations of the Landau level DOS of a 2DEG have recently been carried out for a smooth disorder of arbitrary correlation, based on a diagrammatic [35] or a path-integral technique [36]. Nevertheless, these theories were developed essentially for the case of weak disorder and, hence, turn out to be accurate, especially for high Landau levels. It is clear that, under strong magnetic fields and low temperatures, the low-energy region is likely to be of more physical interest in comparison to

the high-energy one. Thus, the aim of this paper is to find another version of the semiclassical approach, which may get rid of the assumption of weak disorder and must then be useful for low Landau levels.

In section 2, the calculation of the Landau level DOS of a disordered 2DEG proceeds within a semiclassical non-perturbative path-integral approach to a smooth random field, taking explicitly into account the realistic correlation function of the random field via its average potential and force. To control the validity of the approximation used, a computational method is exactly derived in section 3, which is applicable to any random field. The theory is applied in section 4 to a quantum well where the disorder is caused by modulation doping. Finally, section 5 is devoted to conclusions.

2. Semiclassical non-perturbative approach

2.1. Path-integral formulation of the Landau-level DOS

We are dealing with a 2DEG subjected to a random field and a uniform magnetic field perpendicular to the x–y plane of the 2DEG. The concepts of homogeneity and isotropy are not applicable for an individual realization of the stochastic potential. Instead, a stochastic potential U is said to be macroscopically homogeneous and isotropic if $U'(\boldsymbol{r}) = U(\alpha\boldsymbol{r}+\boldsymbol{a})$ for any orthogonal matrix α and any vector $\boldsymbol{a}$ has the same statistical properties as U, in particular, if all correlation functions of U' are the same as for U. In what follows, we will restrict the discussion to the case when the random field $U(\boldsymbol{r})$ is Gaussian, and it may therefore be completely described by a binary correlation function, defined as

$$W(\boldsymbol{r}_1 - \boldsymbol{r}_2) = \langle U(\boldsymbol{r}_1)U(\boldsymbol{r}_2)\rangle, \tag{1}$$

where the angular brackets $\langle\cdots\rangle$ stand for the averaging over all configurations of the random field, which is assumed macroscopically homogeneous. As usual, the field is characterized by its rms potential γ and correlation length Λ, defined as

$$\gamma^2 = \langle U^2\rangle = W(\boldsymbol{r})|_{r=0} \tag{2}$$

and

$$\Lambda = \gamma/F, \tag{3}$$

with the rms force given by

$$F^2 = \langle(\nabla U)^2\rangle = \nabla_1\nabla_2 W(\boldsymbol{r}_1 - \boldsymbol{r}_2)|_{r_1=r_2}. \tag{4}$$

It is well known that the electronic DOS per unit area can be expressed in terms of a Fourier transform of the Green function as

$$\rho(E) = \frac{1}{\pi\hbar}\int_{-\infty}^{+\infty} \mathrm{d}t\ \exp(\mathrm{i}Et/\hbar)\langle G(t)\rangle, \tag{5}$$

with the spin degeneracy included. Here $\langle G(t)\rangle = \langle G(\boldsymbol{r}, \boldsymbol{r}; t)\rangle$ denotes the diagonal part of the averaged Green function, which describes the one-particle properties of a 2DEG in the presence of random and magnetic fields. This is obviously independent of the spatial coordinate in the x–y plane by virtue of the macroscopic homogeneity of the system.

For a particle with effective subband mass m and charge $-e$ (where e is the absolute value of the electron charge) in a magnetic field B, the length and energy scales are given by

$$l_\mathrm{c} = \sqrt{\hbar/eB} \tag{6}$$

and

$$\hbar\omega_\mathrm{c} = \hbar(eB/m). \tag{7}$$

The quantities l_c and ω_c are known as the magnetic length and cyclotron frequency, respectively. These allow us to decide whether a parameter is small, intermediate or large.

A magnetic field pointing along the z direction can be described in the symmetric gauge by a vector potential in the x–y plane $\boldsymbol{A}(\boldsymbol{r}) = (-By/2, Bx/2)$. For a Gaussian random field, the averaged Green function may then be written in terms of a Feynman path integral as [22, 24, 30]

$$\langle G(t)\rangle = \frac{m}{2\pi \mathrm{i}\hbar t}\frac{1}{\mathcal{N}}\oint \mathcal{D}\boldsymbol{r}(\tau)\exp\left\{\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau\left[\frac{m}{2}\dot{\boldsymbol{r}}^2(\tau)+\frac{m\omega_c}{2}(x\dot{y}-y\dot{x})\right]\right.$$
$$\left.-\frac{1}{2\hbar^2}\int_0^t \mathrm{d}\tau\int_0^t \mathrm{d}\tau'\, W[\boldsymbol{r}(\tau)-\boldsymbol{r}(\tau')]\right\}, \tag{8}$$

in which $\mathcal{D}\boldsymbol{r}(\tau)$ denotes the Feynman measure on the set of closed orbits in the plane of the 2DEG and $\mathcal{N}$ is the normalization constant:

$$\mathcal{N} = \oint \mathcal{D}\boldsymbol{r}(\tau)\exp\left\{\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau\,\frac{m}{2}\dot{\boldsymbol{r}}^2(\tau)\right\}. \tag{9}$$

Thus, with the help of a Feynman path integral the DOS of a 2DEG subjected to a perpendicular magnetic field and a random field is represented in terms of the correlation function $W(\boldsymbol{r})$, whose form is specified by the interaction of the 2DEG with disorder and, hence, depends on the system geometry as well as the disorder origin [2, 17]. Equations (5) and (8) set up a basis for discussion of the disorder effect on the Landau level DOS.

2.2. Averaged one-particle Green function

Hereafter, we assume that the random field is varying slowly on the average in space, which crudely implies that

$$\Lambda \gg l_c. \tag{10}$$

The random field may then be dealt with within a semiclassical approach. The derivation follows the same line as in the case treated in [34] for a disordered 2DEG in the absence of a magnetic field. Under the condition (10), the dominant contribution to the path integral entering equation (8) comes from classical orbits of the particle affected by both magnetic and random fields. Furthermore, on these trajectories the correlation function $W(\boldsymbol{r})$ varies slowly. For orbits of relatively small size, we acquire a small quantity of the form

$$\frac{\gamma^2 t^2}{2\hbar^2}-\frac{1}{2\hbar^2}\int_0^t \mathrm{d}\tau\int_0^t \mathrm{d}\tau'\, W[\boldsymbol{r}(\tau)-\boldsymbol{r}(\tau')] \ll 1. \tag{11}$$

This enables the exponential in equation (8) to be expanded in its powers. As a result, we may get an approximation for the averaged Green function:

$$\langle G(t)\rangle = G_0(t)\exp\left(-\frac{\gamma^2 t^2}{2\hbar^2}\right)\left[1+\frac{\gamma^2 t^2 - J(t)}{2\hbar^2}\right], \tag{12}$$

where $G_0(t)$ is the Green function in the presence of a magnetic field but in the absence of disorder

$$G_0(t) = \frac{m}{2\pi \mathrm{i}\hbar t}\frac{\omega_c t/2}{\sin \omega_c t/2} \tag{13}$$

and $J(t)$ is defined by

$$J(t) = \frac{1}{2\pi \mathrm{i} G_0(t)}\frac{1}{\mathcal{N}}\oint \mathcal{D}\boldsymbol{r}(\tau)\exp\left\{\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau\left[\frac{m}{2}\dot{\boldsymbol{r}}^2(\tau)+\frac{m\omega_c}{2}(x\dot{y}-y\dot{x})\right]\right\}$$
$$\times\int_0^t \mathrm{d}\tau\int_0^t \mathrm{d}\tau'\, W[\boldsymbol{r}(\tau)-\boldsymbol{r}(\tau')]. \tag{14}$$

It has been demonstrated [37] that, for equation (13) to make sense at any time, one must replace $\omega_c \to \omega_c - i\eta$ with an infinitesimal $\eta > 0$. It is worth noting that the first and second terms inside the square brackets on the right-hand side of equation (12) refer to the classical DOS and its quantum correction, respectively.

We have now to evaluate the path integral $J(t)$. The correlation function appearing in equation (14) is to be replaced with its Fourier transform, defined by

$$W(\boldsymbol{r}) = \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} \exp(i\boldsymbol{k}\boldsymbol{r}) W(\boldsymbol{k}). \tag{15}$$

The path integral is then exactly performed, yielding

$$J(t) = t^2 \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} W(\boldsymbol{k}) \int_0^1 d\sigma \, \exp\left\{-i\frac{\hbar\boldsymbol{k}^2 t}{2m} \frac{\sin(\sigma\omega_c t/2)\sin[(1-\sigma)\omega_c t/2]}{(\omega_c t/2)\sin(\omega_c t/2)}\right\}. \tag{16}$$

Next, it is clearly observed from equations (5) and (12) that the main contribution to the semiclassical DOS results from such a time region that

$$\gamma t/\hbar \lesssim 1, \tag{17}$$

with γ the rms disorder potential. Further, the $\boldsymbol{k}$ integral in equation (16) is apparently extended primarily over such a wavevector region that

$$|\boldsymbol{k}| \lesssim 1/\Lambda, \tag{18}$$

with Λ the disorder correlation length. Upon combining inequalities (17) and (18), we are in a position to estimate the upper limit of the variable of the exponential in equation (16):

$$\frac{\hbar\boldsymbol{k}^2 t}{2m} \lesssim \frac{\hbar^2/2m\Lambda^2}{\gamma}. \tag{19}$$

In accordance with the semiclassical nature of the random field, we may as usual adopt the following inequality:

$$\frac{\hbar^2/2m\Lambda^2}{\gamma} \ll 1. \tag{20}$$

Upon employing a Taylor series in powers of the small quantity $\hbar\boldsymbol{k}^2 t/2m$ for the exponential in equation (16) truncated after the first order, we are able to obtain for $J(t)$ an approximate expansion:

$$J(t) = \gamma^2 t^2 - i\frac{\hbar F^2 t}{m\omega_c^2}\left(1 - \frac{\omega_c t}{2}\cot\frac{\omega_c t}{2}\right). \tag{21}$$

Lastly, by inserting equation (21) back into (12), we may immediately arrive at a simple expression for the averaged Green function describing the 2DEG moving in a high perpendicular magnetic field and a random field of long-range correlation:

$$\langle G(t)\rangle = G_0(t)\exp\left(-\frac{\gamma^2 t^2}{2\hbar^2}\right)\left[1 + i\frac{\hbar F^2 t}{2m\omega_c^2}\left(1 - \frac{\omega_c t}{2}\cot\frac{\omega_c t}{2}\right)\right]. \tag{22}$$

The rms potential and force of the random field figuring in equation (22) are now rewritten in terms of the Fourier transform of the correlation function as

$$\gamma^2 = \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} W(\boldsymbol{k}) \tag{23}$$

and

$$F^2 = \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} \boldsymbol{k}^2 W(\boldsymbol{k}). \tag{24}$$

2.3. Analytic solution for the Landau level DOS

Let us now return to the calculation of the Landau level DOS. Upon putting equation (22) into (5) with the subsequent use of a spectral expansion of the disorder-free Green function:

$$G_0(t) = \frac{\omega_c}{2\pi}\sum_{n=0}^{\infty}\exp(-\mathrm{i}E_n t), \tag{25}$$

with $E_n = \hbar\omega_c(n+1/2)$ as the Landau levels, we get a corresponding expansion for the DOS:

$$\rho(E) = \frac{\omega_c}{2\pi^2}\sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}\mathrm{d}t\,\exp\left(\frac{\mathrm{i}}{\hbar}(E-E_n)t - \frac{\gamma^2 t^2}{2\hbar^2}\right)\left[1+\mathrm{i}\frac{\hbar F^2 t}{2m\omega_c^2}\left(1-\frac{\omega_c t}{2}\cot\frac{\omega_c t}{2}\right)\right]. \tag{26}$$

Next, the $\cot(\omega_c t/2)$ in equation (26) is to be replaced with a Fourier series. The t integrals appearing are then straightforward by means of [38]:

$$\int_{-\infty}^{+\infty}\mathrm{d}x\,x^n\exp(-px^2-qx) = \left(\frac{\mathrm{i}}{2}\right)^n\frac{\sqrt{\pi}}{p^{(n+1)/2}}\exp\left(\frac{q^2}{4p}\right)H_n\left(\frac{\mathrm{i}q}{2\sqrt{p}}\right), \qquad [\mathrm{Re}\ p>0], \tag{27}$$

with $H_n(x)$ being a Hermite polynomial. Consequently, we are able to represent the DOS of interest in terms of the following series:

$$\rho(E) = n_{\mathrm{LL}}\sum_{n=0}^{\infty}\frac{1}{\sqrt{2\pi}\gamma}\exp\left(-\frac{(E-E_n)^2}{2\gamma^2}\right)\left\{1+\frac{\hbar}{4m\omega_c\Lambda^2}\left[1-2\frac{E-E_n}{\hbar\omega_c}-\frac{(E-E_n)^2}{\gamma^2}\right.\right.$$
$$\left.\left.+2\sum_{k=1}^{\infty}\exp\left(-\frac{(k\hbar\omega_c)^2}{2\gamma^2}+\frac{k\hbar\omega_c(E-E_n)}{\gamma^2}\right)\left(1-\frac{(E-E_n-k\hbar\omega_c)^2}{\gamma^2}\right)\right]\right\}, \tag{28}$$

where $n_{\mathrm{LL}} = 1/\pi l_c^2 = eB/\pi\hbar$ is the degeneracy of a Landau level. The series thus obtained can be rearranged. As a result, we may finally find an analytic solution for the Landau level DOS of a 2DEG in the presence of a perpendicular magnetic field and a semiclassical random field:

$$\rho(E) = n_{\mathrm{LL}}\sum_{n=0}^{\infty}\frac{1}{\sqrt{2\pi}\gamma}\exp\left(-\frac{(E-E_n)^2}{2\gamma^2}\right)$$
$$\times\left\{1+\frac{1}{2m\omega_c^2\Lambda^2}\left[E_n-(E-E_n)-E_n\frac{(E-E_n)^2}{\gamma^2}\right]\right\}. \tag{29}$$

It should be stressed that the above expression is derived under the condition of smoothness of the random field, however, not of its weakness. The DOS in a form analogous to equation (29) was supplied previously [31] but only for a Gaussian choice of the correlation function $W(\boldsymbol{r}) = \gamma^2\exp(-r^2/L^2)$ with $r = |\boldsymbol{r}|$, by means of an expansion of the Green function (8) in the inverse correlation length L^{-1}.

Equation (29) evidently indicates that our DOS describes, as expected, a series of broadened Landau levels. Moreover, the semiclassical DOS is made from two contributions: the purely classical component and its quantum correction, corresponding to the first and second terms inside curly braces in equation (29), respectively. The former exhibits a symmetric Gaussian DOS whose linewidth is determined merely by the disorder potential γ, independent of the correlation length and the applied magnetic field. In contrast, the latter results in an asymmetric non-Gaussian DOS whose broadening depends not only on the potential γ and correlation length Λ (or, equivalently, force F) of the random field but the magnetic field B as

well. It is worth remarking that the role of the quantum correction and, hence, the deviation of the Landau level DOS from the Gaussian form is inversely proportional to the effective mass, the square of the correlation length and the magnetic strength, i.e. to $1/m\Lambda^2B^2$. This implies that the shorter the disorder correlation length and the weaker the magnetic field is, the more non-Gaussian the DOS becomes. Moreover, the quantum correction may be significant for a light particle. Interestingly, the contribution of each Landau level to the integrated DOS is equal to n_{LL} both for the classical limit and the full semiclassical solution. Thus, the quantum correction does not change the spectral weight of a Landau level. We will return to this point in the next section.

It is interesting to note that a semiclassical perturbative expansion of the one-particle Green function (8) in the small ratio l_c/Λ leads, with a Gaussian choice of the correlation function, to an analytic solution to the Landau level DOS [28]. Thereby, the quantum correction is, however, found to be independent of the correlation length. Moreover, such an expansion is proved not to be useful for the calculation of a two-particle Green function defining, e.g., the conductivity of the 2DEGs. An analytic solution may be obtained for the DOS broadened by a more general (Poisson's) distribution of long-range scatterers with a Gaussian potential [29]. However, this is derived simply for the purely classical component of the lowest Landau level DOS by an expansion of the partition function in which both the ratios l_c/Λ and $\gamma/\hbar\omega_{\mathrm{c}}$ are assumed small.

2.4. Conditions of applicability of the semiclassical non-perturbative approach

We now assess the validity of the Landau level DOS (29). In the earlier theories [35, 36] the discussion of the quality of the adopted approximation has been focused mainly on the relation in the lengths involved. Nevertheless, this is obviously decided by the scales not only in the length, but in the strength of the magnetic and random fields, and the electron energy as well. For a fixed energy range $-\infty < E \leqslant E_n$, the conditions are

$$2\Lambda^2 \gg l_{\mathrm{c}}^2\left(n+\tfrac{1}{2}\right) \tag{30}$$

and

$$8\gamma^2\Lambda^2 \gg (\hbar\omega_{\mathrm{c}})^2 l_{\mathrm{c}}^2\left(n+\tfrac{1}{2}\right). \tag{31}$$

The above inequalities and the inequality (20) reveal that the DOS (29) becomes a better approximation when increasing the correlation length Λ and the rms potential γ of the random field, and decreasing the Landau level index n. This means that the disorder is smoother and stronger, e.g., due to heavy doping and the electron energy is lower. The first inequality replaces the condition (10). This requires that the correlation length has to be much larger than the radius of the cyclotron orbit related to the nth Landau level, i.e. its spatial extension, given by $R_{\mathrm{c}}(E_n) = l_{\mathrm{c}}\sqrt{2(n+1/2)}$. In particular, this implies a high enough magnetic field. The second inequality must be fulfilled in order to avoid a negative DOS between two Landau levels, which implies a low enough magnetic field. Thus, both the conditions can be satisfied simultaneously in a moderate range of magnetic fields. These also imply that, in the case of relatively weak disorder, our analytic solution is useful for low Landau levels (see section 3.2). Indeed, because the radius of a particle trajectory is reduced with decreasing the energy (or n), at low energies it is of small size, so that the estimation (11) is well justified. In addition, equation (29) goes over into a Gaussian DOS of the linewidth γ in the limit $\gamma\Lambda \to \infty$, which is the condition (31).

As already mentioned in section 1, for a disorder of arbitrary correlation there has been given in the literature a version of the semiclassical theory of the Landau level DOS with the help of a path-integral technique, provided in [36]. There, the disorder is, however, assumed

to be not only smooth, but weak as well. Then, a semiclassical perturbative path-integral approach may be developed. The basic idea of the method is outlined as follows [36, 39]. The smoothness condition (10) enables the Green function (8) to be expressed as a sum over all classical orbits, in which each (orbit-dependent) term is the product of a classical amplitude and a phase factor including the classical action. Furthermore, under the weakness condition, given by [36]

$$\omega_c \tau_{tr} \gg 1, \tag{32}$$

with τ_{tr} the transport time, the disorder influence on classical orbits and their classical amplitude is negligibly small. The dominant disorder effect on the Green function comes from the shifts in phases due to the modification of the classical actions along the unperturbed trajectories. The latter are to be chosen as saddle-point orbits in the absence of disorder, i.e. within the stationary-phase approximation. As a result, the Landau level DOS is obtained in the following form [36]:

$$\rho(E) = n_{LL} \sum_{n=0}^{\infty} \frac{1}{\sqrt{2\pi}\Gamma(E)} \exp\left(-\frac{(E-E_n)^2}{2\Gamma^2(E)}\right). \tag{33}$$

Here, the linewidth is fixed by

$$\Gamma^2(E) = \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} W(\boldsymbol{k}) J_0^2[kR_c(E)], \tag{34}$$

where $k = |\boldsymbol{k}|$, $J_0(x)$ is the Bessel function of zero order and

$$R_c(E) = l_c\sqrt{2E/\hbar\omega_c} \tag{35}$$

is the radius of the cyclotron orbit as an increasing function of energy. The DOS expression determined by equations (33)–(35) was already derived within perturbation theory with the use of a diagrammatic technique [35]. It was indicated that this holds for high Landau levels and obviously represents a system of Gaussian peaks with the broadening decreasing with energy as $E^{-1/4}$.

Next, we turn to the discussion of the condition for a random field to be weak. For a correlation function depending on the distance only, $W(\boldsymbol{r}) = W(r)$, the transport time entering inequality (32) is specified by [36]

$$\frac{1}{\tau_{tr}} = -\frac{1}{m^2\omega_c^3 R_c^3(E)} \int_0^{\infty} \frac{dr}{r} \frac{dW(r)}{dr}. \tag{36}$$

For simplicity, the integral in equation (36) is estimated for the case of a Gaussian-correlated disorder (with the correlation length $\Lambda = L/2$) to be

$$\frac{1}{\tau_{tr}} = \frac{\sqrt{\pi}}{2} \frac{\gamma^2}{m^2\Lambda\omega_c^3 R_c^3(E)}. \tag{37}$$

Upon replacing $R_c(E)$ in equation (37) by equation (35) and then inserting the transport time appearing into inequality (32), the latter becomes

$$\frac{4}{\sqrt{\pi}} \frac{\hbar\omega_c E^{3/2}}{\gamma^2\sqrt{\hbar^2/2m\Lambda^2}} \gg 1, \tag{38}$$

which must also hold qualitatively for any weak disorder.

The inequality just obtained reveals that, although the condition for validity of the existing theories [35, 36] about the length scale is the same as in our theory, the conditions about the disorder strength and electron energy scales are clearly seen in opposition to the ones from which the formula (29) is derived. The DOS (33) becomes a better approximation when

increasing the correlation length Λ, but decreasing the rms potential γ and elevating the energy E. The increase of the magnetic field B also favours the approximation. According to equation (34), the linewidth of Landau levels $\Gamma(E)$ tends to the rms potential γ in the limit of ultra-long-range correlation $\Lambda \to \infty$, independent of γ. Hence formula (33) fails for low Landau levels in the case where the product $\gamma\Lambda$ is large but Λ is moderate. Thus, the earlier DOS is accurate for the part of the energy spectrum related to high Landau levels, but does not give the correct behaviour for low Landau levels, especially in the limit $\gamma \to \infty$ and Λ fixed. This also is appropriate for light doping. In this sense our DOS, of which low Landau levels and/or heavy doping are in favour, is a complement to the result obtained in [35, 36]. On the other hand, the low-energy region is likely to be of more physical interest in comparison to the high-energy one. Indeed, under a strong magnetic field and low temperature the DOS values at energies less than the Fermi level play a key role, which is normally located within a region of a few cyclotron energies, reckoned from the subband edge [10, 16]. In addition, it is to be noted that, in the intermediate range of parameters, both the DOS formulae give nearly identical results.

The energy regions in question suggest corresponding approximations. Indeed, according to equation (35), the high Landau levels refer to classical orbits of large radius. In that case, one may neglect their (relative) deformation caused by the random force and adopt the stationary-phase approximation, keeping only saddle-point orbits in the classical-path sum for the Green function [36, 39]. Moreover, it was pointed out [35] that the broadening of a Landau level is then of purely classical origin, caused simply by its Gaussian fluctuations due to the random potential, resulting in a Gaussian shape of the DOS, as indicated by equation (33). In contrast, in the low-energy region the cyclotron orbits are of small size and their deformation may become remarkable, in particular for a particle of small effective mass. Therefore, one must take adequate account of the non-Gaussian quantum correction, which is closely connected with the random force and is of greater importance for a light particle, as shown in equation (29).

3. Computational approach

3.1. Reformulation for the Landau level DOS

To develop a computational approach to the DOS of interest, we start from its spectral representation, defined by

$$\rho(E) = 2\left\langle \sum_{\lambda} \varphi_\lambda(\boldsymbol{r})\delta(E_\lambda - E)\varphi_\lambda^*(\boldsymbol{r}) \right\rangle, \tag{39}$$

where E_λ and φ_λ describe the eigenstates of the Hamiltonian $\hat{H}$ for an electron moving in a random potential and a magnetic field. In the asymmetric Landau gauge with a vector potential for the magnetic field $\boldsymbol{A}(\boldsymbol{r}) = (0, Bx)$, the explicit form of the Hamiltonian in real space is

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{e^2B^2}{2m}x^2 + \frac{eB}{m}x\frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial y} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2} + U(x, y) = \hat{H}_0 + U(x, y), \tag{40}$$

where $\hat{H}_0$ is the Hamiltonian with a magnetic field but without a random potential. It is convenient both for the analytical and computational treatments to employ Born–von Kármán cyclic boundary conditions for the y direction with normalization length L_y. Hereafter, in all equations the limit $L_y \to \infty$ is implicitly assumed.

Equation (39) requires the numerical solution of an eigenvalue problem and the statistical average over many realizations of the stochastic potential. This is not feasible, as the dimension of the matrix is typically of the order of a few million. Therefore, we derive a formulation,

which is self-averaging, so that $\rho(E)$ can be determined from *one* realization of the stochastic potential U. Furthermore, we reformulate the eigenvalue problem as an initial-value problem and use a discretization of the differential operator in real space. Then the time-dependent Schrödinger equation is solved numerically, taking fully into account the sparsity of the matrix.

The Green function appearing in equation (5), which follows from equation (39), obviously takes the following form [37]:

$$G(\boldsymbol{r}, \boldsymbol{r}, t) = (\boldsymbol{r}|\mathrm{e}^{-\mathrm{i}\hat{H}t/\hbar}|\boldsymbol{r}), \tag{41}$$

where the round brackets $(|\cdots|)$ stand for the expectation value of an operator.

In principle, $\langle G(t)\rangle = \langle G(\boldsymbol{r}, \boldsymbol{r}, t)\rangle$ can be evaluated by computationally solving equation (41) and averaging over thousands of realizations of the disorder potential $U(\boldsymbol{r})$. The task can be simplified by exploiting the self-averaging properties of the random field [40]. For this purpose, we replace the ensemble average by an average over the y coordinate and employ the conservation of trace:

$$\begin{aligned}\langle G(t)\rangle &= \frac{1}{L_y}\int_{-L_y/2}^{+L_y/2} \mathrm{d}y (x=0, y|\mathrm{e}^{-\mathrm{i}\hat{H}t/\hbar}|x=0, y) \\ &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{d}k_y (x=0, k_y|\mathrm{e}^{-\mathrm{i}\hat{H}t/\hbar}|x=0, k_y).\end{aligned} \tag{42}$$

The wavefunction which belongs to the quantum number k_y is $\exp(\mathrm{i}k_y y)/\sqrt{L_y}$. With the help of the identity

$$\mathrm{e}^{-\mathrm{i}k_y y}\hat{H}_0(\boldsymbol{r}, -\mathrm{i}\hbar\nabla)\mathrm{e}^{+\mathrm{i}k_y y}\cdots = \hat{H}_0(\boldsymbol{r} + (\hbar k_y/eB)\boldsymbol{e}_x, -\mathrm{i}\hbar\nabla)\cdots, \tag{43}$$

and the translational invariance of U (see section 2.1), the case of $k_y \neq 0$ can be reduced back to $k_y = 0$. At the same time, the x coordinate is shifted by $\hbar k_y/eB$ and the averaged Green function becomes

$$\langle G(t)\rangle = \frac{eB}{2\pi\hbar}\int_{-\infty}^{+\infty} \mathrm{d}x (x, k_y=0|\mathrm{e}^{-\mathrm{i}\hat{H}t/\hbar}|x, k_y=0). \tag{44}$$

The integration over x can be replaced by the trace over any complete set of eigenstates. Here, we use the unperturbed eigenfunctions for $k_y = 0$, known as

$$\varphi_n(x) = \frac{1}{\sqrt{2^n\sqrt{\pi}n!l_\mathrm{c}}}\exp\left(-\frac{x^2}{4l_\mathrm{c}^2}\right)H_n\left(\frac{x}{l_\mathrm{c}}\right), \tag{45}$$

with $n = 0, 1, 2, \ldots$. Then the final expression for the averaged Green function is

$$\langle G(t)\rangle = \frac{eB}{2\pi\hbar}\sum_{n=0}^{\infty}(n, k_y=0|\mathrm{e}^{-\mathrm{i}\hat{H}t/\hbar}|n, k_y=0) \tag{46}$$

and the matrix elements of the Hamiltonian take the form

$$\hat{H}_{nn'} = \hbar\omega_\mathrm{c}\left(n+\frac{1}{2}\right)\delta_{nn'} - \mathrm{i}\hbar\omega_\mathrm{c}X_{nn'}\frac{\mathrm{d}}{\mathrm{d}y} - \frac{\hbar^2}{2m}\delta_{nn'}\frac{\mathrm{d}^2}{\mathrm{d}y^2} + U_{nn'}(y), \tag{47}$$

in which

$$X_{nn'} = \begin{cases} \frac{1}{2}l_\mathrm{c}\sqrt{n+n'+1}, & \text{for } |n-n'|=1, \\ 0, & \text{otherwise} \end{cases} \tag{48}$$

and

$$U_{nn'}(y) = \int_{-\infty}^{+\infty}\mathrm{d}x\,\varphi_n^*(x)U(x, y)\varphi_{n'}(x). \tag{49}$$

For $U = 0$, the solution for the disorder-free Green function $G_0(t)$ (25) is reproduced, which leads to the DOS in the absence of a random field:

$$\rho_0(E) = \frac{eB}{\pi\hbar}\sum_{n=0}^{\infty}\delta\left[E - \hbar\omega_c\left(n + \frac{1}{2}\right)\right]. \tag{50}$$

In the limit $B \to 0$, the latter expression changes over into a step function:

$$\rho_0(E, B = 0) = \frac{m}{\pi\hbar^2}\theta(E), \tag{51}$$

known as the DOS of a 2DEG in the absence of both random and magnetic fields.

By virtue of the Fourier transform (5), the integrated DOS including the spin degeneracy is equal to $2G(t = 0)$, and it follows clearly from equation (46) that the spectral weight of each Landau level is equal to n_{LL}, independently of the random field.

The Gaussian disorder potential can be generated as a Fourier series with random phases [33, 41]:

$$U(\boldsymbol{r}) = \frac{1}{\sqrt{L_x L_y}}{\sum_{\boldsymbol{k}}}' e^{i\boldsymbol{k}\cdot\boldsymbol{r}} C(\boldsymbol{k})\sqrt{W(\boldsymbol{k})}, \tag{52}$$

where the prime over the summation means that the point $\boldsymbol{k} = 0$ is to be excluded and

$$C(\boldsymbol{k}) = e^{i\Phi(\boldsymbol{k})} \tag{53}$$

with the random phases $\Phi(\boldsymbol{k})$ distributed independently and uniformly in the interval $[-\pi, +\pi)$. The reality of $U(\boldsymbol{r})$ requires the constraint $\Phi(\boldsymbol{k}) = -\Phi(-\boldsymbol{k})$. The potential is periodic in x and y directions with periods L_x and L_y, respectively. In the limit $L_x, L_y \to \infty$ it is macroscopically homogeneous and isotropic. The correlation function (1) follows from the relation $\langle C(\boldsymbol{k}_1)C(\boldsymbol{k}_2)\rangle = \delta_{\boldsymbol{k}_1,-\boldsymbol{k}_2}$.

The expression (46) is evaluated computationally by solving an initial-value problem and the DOS (5) is calculated by means of a fast Fourier transform. The method is described in [41].

3.2. Examples

For the numerical calculations, we use dimensionless quantities, defined by $\hbar = l_c = \omega_c = 1$. In addition, the DOS is normalized by $\rho^* = m/\pi\hbar^2$, which is the DOS of a two-dimensional gas of free electrons (51). It is observed that in the classical limit (30) the coupling of different Landau levels is small so that only a finite number $|n - n'| \leqslant n_{off}$ of off-diagonal elements $U_{nn'}$ has to be taken into account. Furthermore, the sum in equation (46) can be restricted to a finite number of Landau levels n_{max}, which needs to be only somewhat larger than the upper limit of the argument E. For the computational solution, we used $n_{off} = 2$, $n_{max} = 15$ for an energy range $E \leqslant 10$. The normalization length L_y has to be much larger than the correlation length Λ, in order to ensure self-averaging. For the present parameters a value of $L_y = 32\,000$ was sufficient. We checked that the results were independent of L_y and that different realizations of the potential, given by different sets of random numbers, lead to the same result for the DOS. For technical reasons, the stochastic potential (52) is also periodic in the x direction. Here, it is sufficient to choose the normalization length to be somewhat larger than the extension of the highest Landau level in the x direction, as given by the distance between the classical points of return $2\sqrt{2n_{max}-1}$. In the calculation, we used $L_x = 16$. The differential operators in equation (47) were discretized using second-order finite differences with a mesh size $\Delta y = 1/4$. With this set of parameters we found full convergence and we shall hereafter refer to the computational solution as the exact DOS.

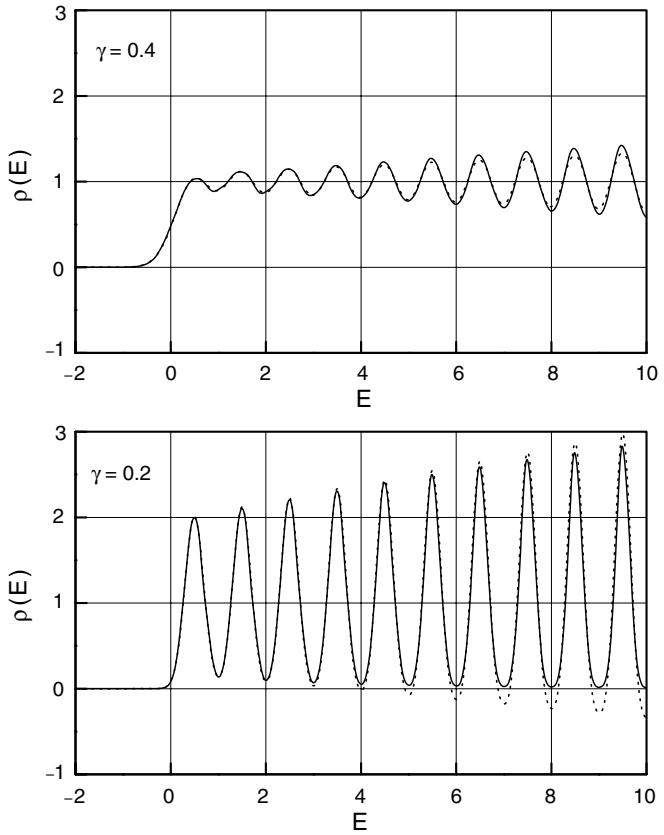

Figure 1. Landau level DOS $\rho(E)$ versus energy E in dimensionless units for a Gaussian correlation function $W(\boldsymbol{r}) = \gamma^2 \exp(-r^2/L^2)$ with the disorder parameters $L = 6$, $\gamma = 0.4$ (upper part) and 0.2 (lower part). The full and dotted curves refer to the computational and analytic solutions, respectively.

As a first example, we have evaluated the DOS in the case of a Gaussian correlation function $W(\boldsymbol{r}) = \gamma^2 \exp(-r^2/L^2)$ for $L = 6$ and different values $\gamma = 0.4$ and 0.2. The same function was addressed in [32]. Let us first check the smoothness criteria (20), (30) and (31). The correlation length is equal to $\Lambda = L/2$. For $\gamma = 0.4$, we have $\gamma\Lambda^2 = 3.6$, $2\Lambda^2 = 18$ and $8\gamma^2\Lambda^2 = 11.52$. For $\gamma = 0.2$, we obtain $\gamma\Lambda^2 = 1.8$, $2\Lambda^2 = 18$ and $8\gamma^2\Lambda^2 = 2.88$. From these estimates we expect that, with $\gamma = 0.4$, the analytical formula (29) is a good approximation for $E \leqslant 10$. With $\gamma = 0.2$, the first and third inequalities are hardly fulfilled and we expect negative values for the DOS after a few Landau levels.

The result of the numerical calculations is displayed in figure 1. The computational result (full curve) is compared with the analytic one (dotted curve). Let us first examine the general properties of the exact solution. For the first Landau level, the lineshape is virtually the same as for the classical result (not shown). With increasing energy, the peaks become increasingly narrow, but the shape of the curve is not distinctively different from the Gaussian form, which is, as mentioned in section 2.3, due to the large correlation length in use. In a simplified picture, this line narrowing could be interpreted as an averaging of the random potential with the probability density of the Landau level $|\varphi_n(x)|^2$. Then the linewidth of the nth Landau level should be equal to $\langle U_{nn}\rangle$. It turns out that the linewidth is smaller than this value, which

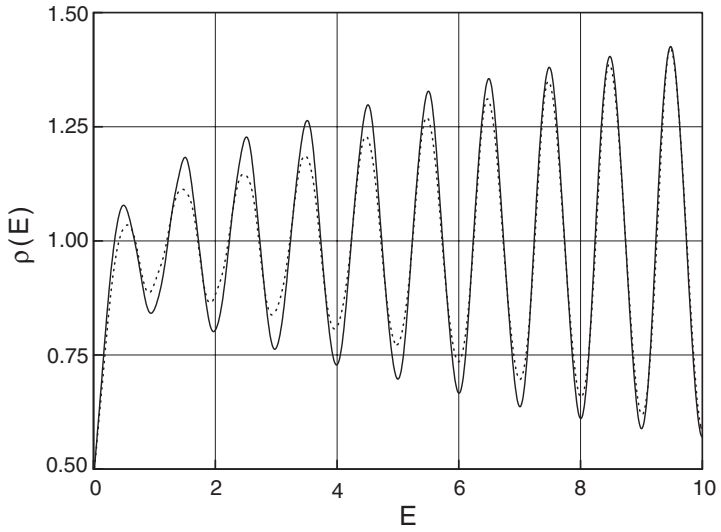

Figure 2. Computational solution for the Landau level DOS $\rho(E)$ versus energy E in dimensionless units for different correlation functions: exponential $W(\boldsymbol{r}) = \gamma^2 \exp(-r/L)$ (full curve) and Gaussian $W(\boldsymbol{r}) = \gamma^2 \exp(-r^2/L^2)$ (dotted curve). The disorder parameters in both cases are $L = 6$ and $\gamma = 0.4$.

means that motional narrowing due to quantum-mechanical motion in the y direction plays an important role. In the whole energy range the minima are located near $E = n$ and the maxima near $E = n + 1/2$. The height of the maxima is about a linear function of the energy. The same is true for the minima, as long as they do not come close to zero. We will see later that the linear scaling of the maxima and minima is not a universal feature.

Next, we check the quality of the analytical solution (29). With $\gamma = 0.4$, the approximation is reasonable in the whole energy range. Negative values for the DOS will occur only at much larger energies ($E \approx 35$). For the first three ($n = 2$) Landau levels, the analytical and computational solutions are virtually indistinguishable. A small quantitative deviation from the exact result is observed for $n \geqslant 3$. With $\gamma = 0.2$, the approximation for the first three Landau levels is also very good. Also, the overall relative error is small. However, starting with $n \geqslant 3$, the analytical DOS becomes negative between two Landau levels (at $E = n$).

Despite a large time consumption, a major advantage of the computational approach is that its validity is independent of a specific choice of the correlation function. As an illustrating example, we have calculated the DOS for an exponential correlation function $W(\boldsymbol{r}) = \gamma^2 \exp(-r/L)$. Then the relevant average force F is, according to equation (24), divergent so that the semiclassical method developed in the preceding section fails to be valid, whereas the computational method works quite well. Further, since $W(\boldsymbol{r})$ is non-analytic at $\boldsymbol{r} = 0$, analytic solutions based upon a Taylor expansion at $\boldsymbol{r} = 0$ derived, e.g., in [32] cannot be used.

The computational DOS for an exponential correlation function with $L = 6$ and $\gamma = 0.4$ is depicted by a full curve in figure 2. For comparison, the corresponding solution for a Gaussian correlation function with the same parameters is indicated by a dotted curve. In both cases, the qualitative behaviour of the DOS is the same regarding the curve narrowing with increasing energy and the positions of the maxima and minima. A difference is observed in the behaviour of the peak heights as a function of energy. For the full curve, the peak heights scale approximately like $\sqrt{E}$, in contrast to the linear scaling for the dotted curve.

4. Quantum well

4.1. Correlation function

We shall now apply the forgoing theory to evaluate the Landau level DOS in a quantum well where the motion of the electrons are confined in the z direction by two infinite potential barriers at its boundaries, $0 \leqslant z \leqslant a$, with a the width of the well. The disorder is normally caused by impurity doping, surface roughness and alloying. As indicated above, the theoretical analysis of the disorder effect on the Landau levels is simply reduced to finding the correlation function for the 2DEG to be treated.

Fluctuations in the density of ionized impurities give rise to a random field. We assume a modulation doping where the impurities are implanted in the system at a distance z_i from a boundary of the quantum well. The correlation function of the random field created by a 2D sheet of impurities was derived in [17] to be

$$W(\boldsymbol{k}) = \left(\frac{Ze^2}{2\epsilon_\mathrm{L}\epsilon_0}\right)^2 \frac{n_\mathrm{i}}{k^2\epsilon^2(k)} F_\mathrm{ei}^2(k). \tag{54}$$

with $k = |\boldsymbol{k}|$. Here n_i denotes the 2D impurity density and Z is the impurity charge. The ϵ_L is the dielectric constant of the background lattice and $\epsilon(k)$ is the dielectric function allowing for the screening of disorder interaction by the 2DEG. The form factor $F_\mathrm{ei}(k)$ takes into account a finite extension of the electron state along the z direction, so that it depends on the geometry of the system, viz. the well width a and the position of the impurity sheet z_i, as follows:

$$F_\mathrm{ei}(k) = \frac{4\pi^2}{ka(4\pi^2 + k^2a^2)} \begin{cases} \mathrm{e}^{kz_\mathrm{i}}(1 - \mathrm{e}^{-ka}), & \text{for } z_\mathrm{i} \leqslant 0, \\ 2 - \mathrm{e}^{-kz_\mathrm{i}} - \mathrm{e}^{-k(a-z_\mathrm{i})} + \dfrac{k^2a^2}{2\pi^2}\sin^2\left(\dfrac{\pi z_\mathrm{i}}{a}\right), & \text{for } 0 \leqslant z_\mathrm{i} \leqslant a. \end{cases} \tag{55}$$

Within the Thomas–Fermi approximation, the dielectric function is supplied by

$$\epsilon(k) = 1 + \frac{k_\mathrm{s}}{k} F_\mathrm{ee}(k), \tag{56}$$

where k_s is the screening wavevector (inverse of the screening length) and $F_\mathrm{ee}(k)$ is the form factor for the electron–electron interaction potential in the 2DEG. The electron–electron interaction is to be modified by the z extension of the electron state, so that

$$F_\mathrm{ee}(k) = \frac{1}{4\pi^2 + k^2a^2}\left(3ka + \frac{8\pi^2}{ka} - \frac{32\pi^4}{k^2a^2}\frac{1 - \mathrm{e}^{-ka}}{4\pi^2 + k^2a^2}\right). \tag{57}$$

A particular problem is the calculation of the screening wavevector in the presence of both disorder and magnetic field. Several authors proposed a formula of the kind [2]

$$k_\mathrm{s} = \frac{\mathrm{e}^2}{2\varepsilon_\mathrm{L}\varepsilon_0}\rho(E_\mathrm{F}). \tag{58}$$

For a given DOS, the Fermi energy E_F implicitly depends on the density of electrons. Thus, the DOS, the Fermi level and the screening wavevector have to be determined self-consistently. Unfortunately, this procedure has a fixpoint, characterized by $\rho(E_\mathrm{F}) = \infty, k_\mathrm{s} = \infty, \varepsilon(k) = \infty$, $U(\boldsymbol{r}) \equiv 0$, and $\rho(E) = \rho_0(E)$, as given by equation (50). As a consequence, the random field created by the charged impurities will always be totally screened. To avoid this spurious solution within the self-consistent Thomas–Fermi screening, one has to use an improved formula for the dielectric function, which has not been available so far. For simplicity, we

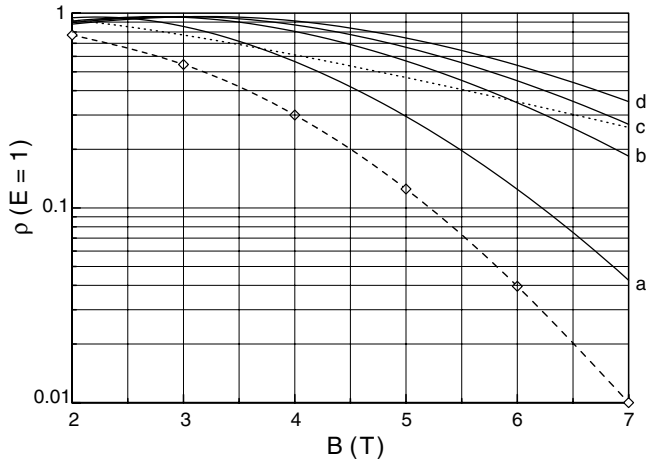

Figure 3. Analytical solution for the Landau level DOS at the cyclotron energy $\rho(E = 1)$ versus magnetic field B for a quantum well of width $a = 140$ Å, doped with $Z = 1$, $z_i = -200$ Å, and various impurity densities $n_i = 0.8$ (a), 1.2 (b), 1.4 (c), and 1.6×10^{12} cm^{-2} (d). The full, broken and dotted curves refer to our calculation, the prediction of [32] (figure 3, diamonds) and the measurement of [4], respectively.

replace $\rho(E_F)$ by the DOS without both random and magnetic fields, $\rho_0(E, B = 0)$, according to equation (51). Then the screening wavevector is equal to

$$k_s = \frac{me^2}{2\pi\varepsilon_L\varepsilon_0\hbar^2}. \tag{59}$$

The principal results are rather insensitive to the special choice of k_s.

4.2. Numerical results and comparison with experiment

The numerical results for the Landau level DOS are to be presented in connection with some available experimental findings. These are specified for a modulation-doped quantum well made from GaAs with the effective mass $m = 0.067\ m_e$. The DOS is measured in units of $\rho^* = 2.8 \times 10^{10}$ meV^{-1} cm^{-2} and the energy in units of $\hbar\omega_c$.

First, we have calculated the DOS at the cyclotron energy ($E = 1$)—between the lowest and first excited Landau levels—as a function of the applied magnetic field. The geometrical parameters are taken from [4], sample 1 for magnetization measurement. The well width is $a = 140$ Å. The layer doped with the impurity charge $Z = 1$ is assumed to be located in the middle of the barrier so that $z_i = -200$ Å. Since the impurity density n_i was not indicated, we perform the calculation with various values $n_i = 0.8$, 1.2, 1.4 and 1.6×10^{12} cm^{-2}. The result thus obtained for the DOS $\rho(E = 1)$ is displayed in dimensionless units for a range of magnetic fields from $B = 2$–7 T by full curves in figure 3. There, the theoretical prediction of [32] (broken curves) and the data of [4] (dotted curve), which were fitted by a Gaussian DOS with a broadening γ (meV) $= \sqrt{B\ (\mathrm{T})}$, are also plotted for comparison. At the values of B used, our results are clearly found in quantitative agreement with the experimental data, both in the order of magnitude of the DOS and its magnetic-field behaviour as well. In particular, the fit is very good at the doping level $n_i = 1.4 \times 10^{12}$ cm^{-2}. In distinction, the exact solution of the simple model based on a quadratic correlation function derived in [32] gives at a large magnetic field a small DOS between the Landau levels. At $B = 7$ T, the predicted DOS is

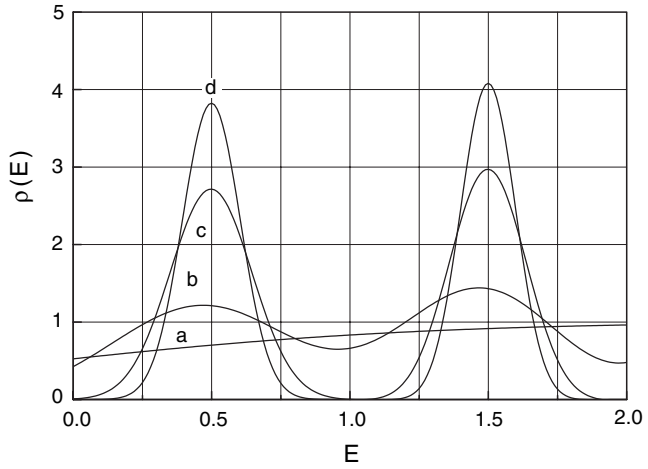

Figure 4. Analytical solution for the Landau level DOS $\rho(E)$ versus energy E in dimensionless units for a quantum well of width $a = 75$ Å, doped with $Z = 1$, $z_i = -200$ Å, under a doping level $n_i = 5 \times 10^{11}$ cm^{-2} and different magnetic fields $B = 1$ (a), 3 (b), 7 (c) and 10 T (d).

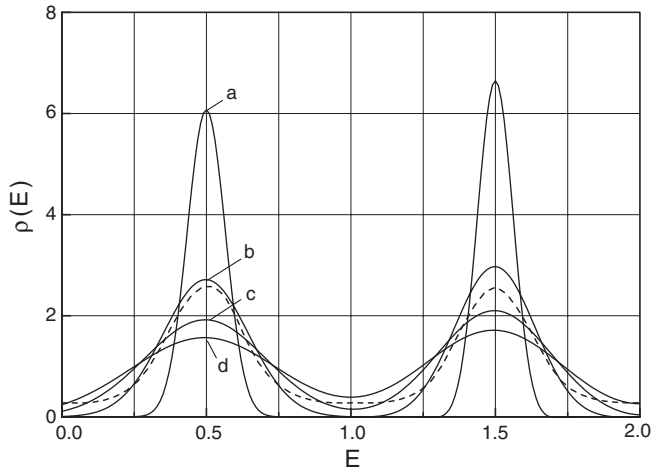

Figure 5. Analytical solution for the Landau level DOS $\rho(E)$ versus energy E in dimensionless units for a quantum well of width $a = 75$ Å, doped with $Z = 1$, $z_i = -200$ Å, under a magnetic field $B = 7$ T and various impurity densities $n_i = 0.1$ (a), 0.5 (b), 1 (c) and 1.5×10^{12} cm^{-2} (d). The full and broken lines refer to our calculation and the measurement of [8], respectively.

smaller than the measured one by up to more than one order of magnitude, even with a best choice of the fitting parameters. Moreover, an inspection of the full curves in figure 3 reveals that the Landau level DOS at the cyclotron energy exhibits a rapid decrease when increasing B (about square exponential) and lowering n_i (about linear exponential), which is expected from the semiclassical solution (29) and the linear n_i dependence of the mean squared potential γ^2.

Next, we evaluated the Landau level DOS of a sample studied in [8] by magnetocapacitance measurement. The parameters are the same as in figure 3, except for the well width $a = 75$ Å. Figures 4 and 5 depict the DOS as a function of energy $\rho(E)$. In figure 4 this is plotted under a doping level $n_i = 5 \times 10^{11}$cm^{-2} and different magnetic fields $B = 1, 3, 7$ and 10 T, whereas

in figure 5 this is under a value of $B = 7$ T and various impurity densities $n_i = 0.1, 0.5, 1.0$ and 1.5×10^{12} cm^{-2}. There, the data of [8] at $B = 7$ T (broken curve) is also reproduced for comparison. Figure 4 illustrates an apparent deviation of the DOS from the Gaussian form at lower magnetic fields. With a reduction of B the Landau levels become more asymmetric and their overlap becomes larger, and at $B = 1$ T the lineshape is almost flat. The apparent non-Gaussian feature of the broadening is due to the significant quantum correction associated with a small effective mass of the material under consideration, as quoted in section 2.3. The structureless behaviour of the Landau level DOS at low magnetic fields was already seen for the case of weakly disordered 2DEGs, based on the SCBA theory with the use of nonlinear self-consistent screening [16, 42]. It is observed from figure 5 that the lineshape calculated with $n_i = 1 \times 10^{12}$ cm^{-2} is in reasonable agreement with the measured one. It has to be mentioned that our theory may provide a remarkable DOS between the Landau levels, whereas the authors of [8] had to invoke a constant background DOS of 28% of ρ^* to fit their data.

5. Conclusions

In summary, in this paper we have proposed two schemes for calculating the Landau level DOS of a 2DEG subjected to a uniform perpendicular magnetic field and a random field of arbitrary correlation. Both of the methods are able to work with the *realistic correlation function* of the latter. This allows us to take complete account of various details of the actual geometry of a 2DEG, such as its extension into the bulk and its 2D screening of the disorder interaction. Also, this allows us to take the actual origin of the disorder and to evaluate the disorder effect on the lineshape due, for example, to impurity doping, surface roughness and alloying. Thus, our theory offers a microscopic description of the Landau level DOS of actual quasi-2DEGs.

It is to be stressed that all existing theories of the Landau level DOS are approximation schemes. Almost all of them were based on simple models of the correlation function with some fitting parameters, and developed different methods for exact or approximate calculation. However, the theoretical prediction may not be in agreement with experiment, even with an exact solution, as in the case of a quadratic correlation function [32]. In contrast, our theory starts from the realistic correlation function and turns out to be able to give a quantitative explanation of experimental data with no fitting parameters.

The analytic theory is established within a semiclassical non-perturbative path-integral approach to the random field and provides a simple closed solution (29) for the Landau level DOS, whereas the computational approach is exactly derived, whose validity is independent of a specific choice of the correlation function. As a consequence, the former is applicable only to a smooth random field, whereas the latter works quite well with any random field. Under the criteria (20), (30) and (31), the analytical solution has been found to fit the computational solution very well.

The semiclassical formula (29) for the Landau level DOS is composed of the classical component and its quantum correction. The former is responsible for a symmetric Gaussian DOS and is associated merely with fluctuations in the disorder potential, whereas the latter is for an asymmetric non-Gaussian DOS and is connected not only with fluctuations both in the potential and force of the random field, but with the applied magnetic field as well. This implies that there is not necessarily a universal form for the Landau level DOS, which can go over from a Gaussian form to another one, depending on the disorder correlation length and the magnetic field. Such a dependence of the DOS behaviour on the correlation length has already been shown in [43] for disordered 2DEGs in the absence of a magnetic field. Under a disorder of rather short correlation length and rather low magnetic field, the deviation from the Gaussian form is found to be significant, especially for a particle of small effective mass, as in GaAs.

Our solution (29) is useful for low Landau levels, whereas the solution (33) due to previous authors [35, 36] is accurate for high Landau levels. It is worth noting that the classical DOS (33) includes the disorder effect associated, via the linewidth (34), with the random potential only, whereas the semiclassical DOS (29) allows for, via the rms potential and force, the influence of both the potential and its spatial derivative of the first order. This suggests that our semiclassical non-perturbative path-integral approach is to be regarded as a first step towards a better approximation, especially for a non-smooth disorder, where one might develop a quantum theory of the broadening of Landau levels with the aid of an expansion involving the spatial derivatives of all orders of the disorder potential. This idea was, in fact, proved in [43] to be successful for disordered low-dimensional electron systems in the absence of magnetic field. In the case of a non-smooth random potential its short-range fluctuations make a key contribution to the energy spectrum. This requires a quantum description of electron states and may likely give rise to an apparently non-Gaussian Landau level DOS.

Another merit of our theory is that it might be applied on an equal footing to calculation of the disorder effect on a two-particle Green function defining, for example, the conductivity of 2DEGs. In contrast, it was shown [36, 39] that the semiclassical perturbative path-integral approach with the use of saddle-point orbits turns out to be unable to produce the relevant disorder effect, so that one has to go beyond the stationary-phase approximation, taking explicitly into account the deformation of classical orbits.

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